Robot Localization

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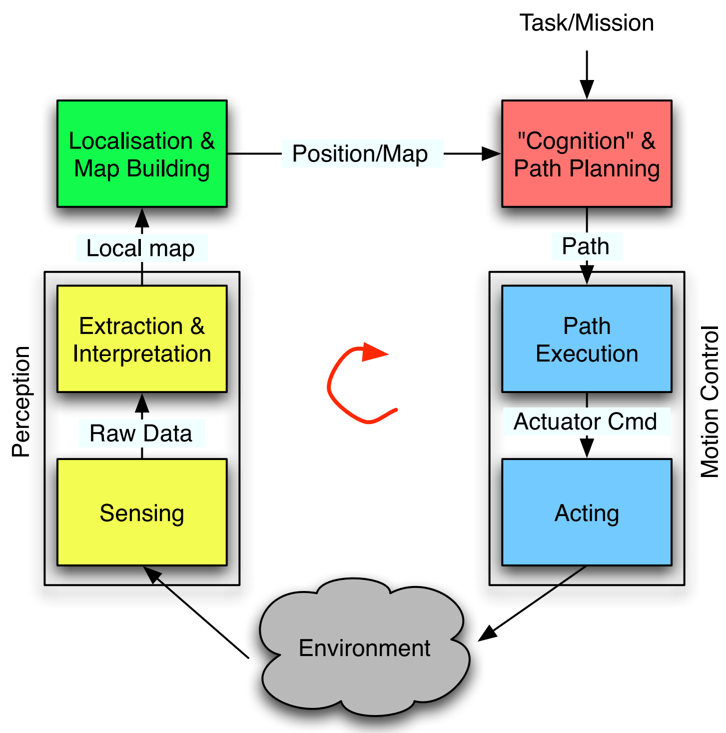
Outline

1. Introduction
2. Taking it apart
3. Representations
4. Features
5. Prediction
6. Matching
7. Position Estimation
   - Topological Pose Estimation
   - Gaussian PDF
   - Monte-Carlo Methods
8. Examples
9. Wrap-up

System Context



Problems in Mobile Robotics

1. Where am I?
2. Where do I need to go?
3. When have I arrived?

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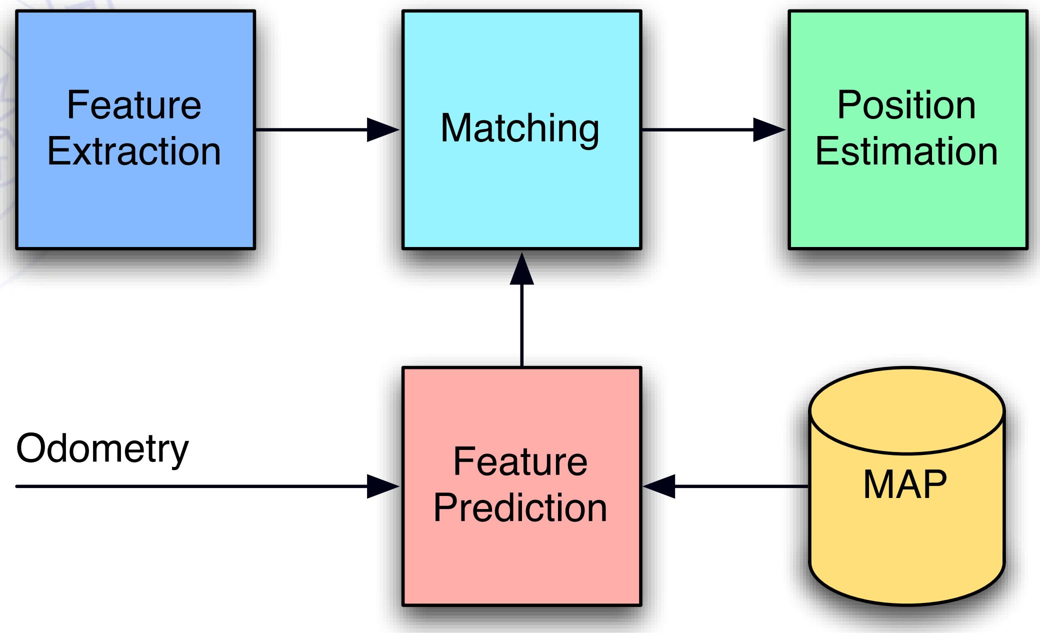
The process



What do we need?

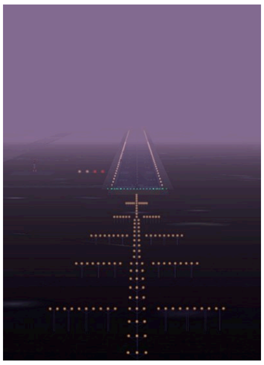
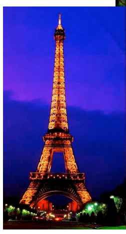
Question:

What do we need to complete the process?

The components

- Map representation
- Selection of features
- A strategy for matching of features to maps
- A way to predict vehicle position
- A way to predict feature locations

Map/Setup

Odometry	Instrumented Environments	High-level features
 Treasure map	 airport light	Elevator  Entrance  Eiffel tower
Unreliable	Expensive	Requires recognition challenging

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Map representations

Question:

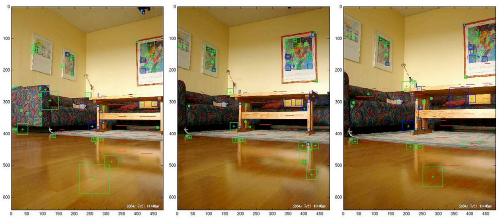
What would be good map representations?

Map representation

- Appearance based maps
- Topological maps
- Grid based maps
- Feature based maps

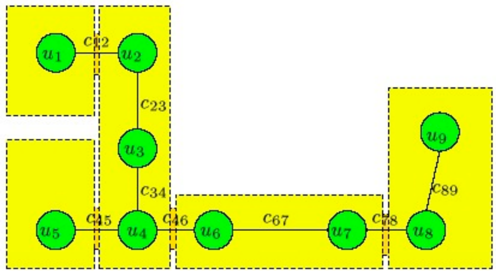
Environmental Maps

Landmarks

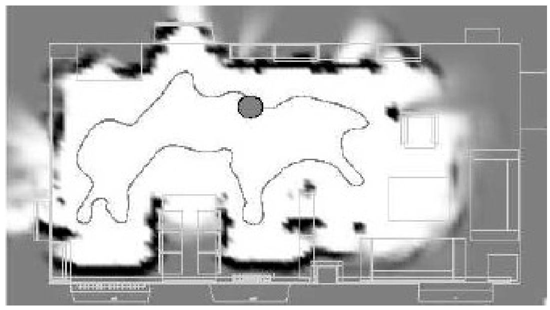


Feature Maps

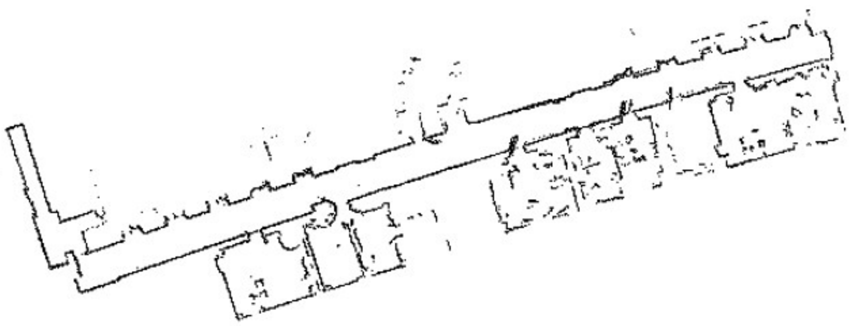
Topological Map



Grid Maps



Appearance based maps



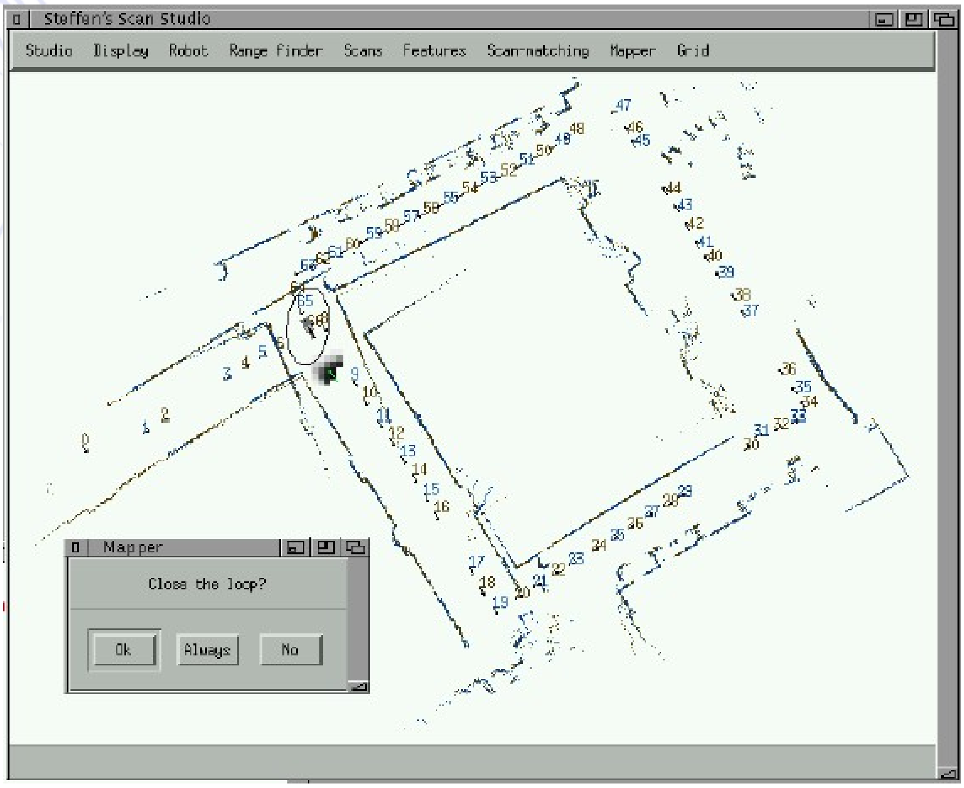
Pros

- Direct alignment of sensor data
- Easy to model

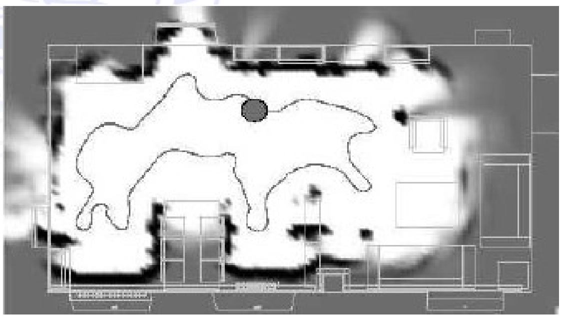
Cons

- Generalizes poorly
- One sensor system
- Ex: ScanStudio by Gutmann

ScanStudio



Gridmaps

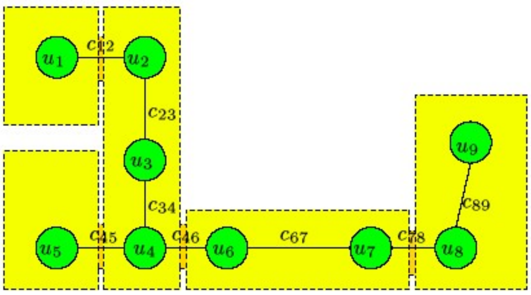


- Easy to understand
- O(env^2) in size
- Easy to update (might be slow)

Feature Maps

- Discrete feature map
- Easy for multi feature intg
- Easy to handle
- O(*features*)

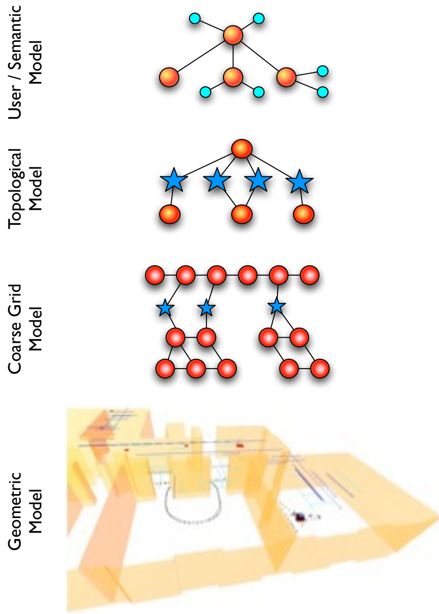
Topological Maps



- Graph representation
- Place recognition
- O(*places*)
- Coarse localisation
- A good planning rep.

Mixed Representations

- Mixed maps are gaining in importance
- Topology for overall layout
- Sub-maps for detailed models
- A way to handle complexity



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Features

- Already covered in a separate lecture(s)
- Which features for which maps?

Method	Grid	Appearance	Feature	Topology
Raw Data	yes	YES		
Points		yes	YES	
Lines			YES	yes
Geometry			YES	yes
Object	yes		YES	YES

Features

- Feature models include
  - ID, Parameters, Uncertainty
- Gaussian Model: $\mathcal{N}(\mu, \Sigma)$
- Generic Sensor Model: $p(z|x)$
- Matching $\leftrightarrow$ Estimation $\leftrightarrow$ Detection

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Pose / Map Prediction

- Initial estimation of new pose based on odometry data
    - Already covered as part of uncertainty modelling
    - Covariance propagation

$$p' = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \Delta s \cos\left(\theta + \frac{\Delta\theta}{2}\right) \\ \Delta s \sin\left(\theta + \frac{\Delta\theta}{2}\right) \\ \frac{\Delta s_r - \Delta s_l}{2l} \end{bmatrix}$$

$$\Sigma_{p'} = \nabla_p f \Sigma_p \nabla_p f^T + \nabla_{\Delta_{rl}} f \Sigma_\Delta \nabla_{\Delta_{rl}} f^T$$

- Propagation of uncertainty to map features
    - Kinematic Update
    - Pose uncertainty as your system noise (Q)

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Matching

- Tied to the map representation
- Grid based - measure correlation/match
- Grid based - voting based matching
- Appearance - voting / scan correlation
- Feature based
  - Nearest neighbor
  - Mahalanobis / Probabilistic
  - Voting based

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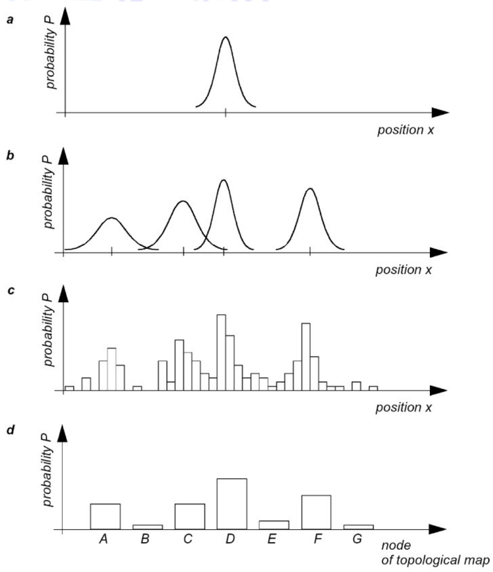
Pose updating – Uncertainty Model



- The selection of an uncertainty model
  - Single hypothesis
  - Sum of Gaussians
  - Probability grid
  - Topological Graph
  - Particle Based

Pose updating - Uncertainty Model

- The selection of an uncertainty model influences the updating methodology
- The uncertainty model is coupled to the environmental representation
- The model influences strongly the computational requirements

Uncertainty Modelling – Markov Approach

- Assume the world is divided into places/states $s \in P$
- Estimation of $p(s_t)$ given s_{t-1} and sensory data z_t
- Formally

$$p(s_t|z_t) = \int p(s_t|s'_{t-1}, z_t)p(s_{t-1})ds'_{t-1}$$

Integration needed as s_t could be reached from multiple locations

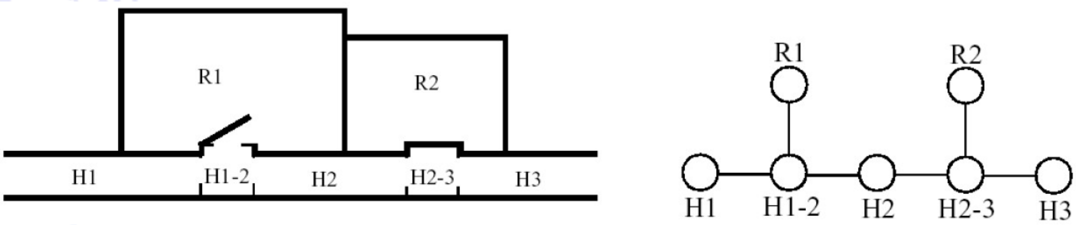
Uncertainty modelling – Markov Approach

- Markov assumption: all knowledge encoded in the pose/state estimate
- There is a probability model for motion updating
- There is a model for $p(z|s)$ i.e. a sensor model, as

$$p(s|z) = \frac{p(z|s)p(s)}{p(z)}$$

  where $p(s)$ is location model and $p(z)$ is the sensor noise model
- These assumptions are relative weak

Topological modelling – dervish example



	Wall	Closed door	Open door	Open hallway	Foyer
Nothing detected	0.70	0.40	0.05	0.001	0.30
Closed door detected	0.30	0.60	0	0	0.05
Open door detected	0	0	0.90	0.10	0.15
Open hallway detected	0	0	0.001	0.90	0.50

Topological modelling – dervish example

- Here the probability updating is used for direct lookup of $p(s|z)$, where s is any of the nodes in the topological map
- As robot moved through environment the graph is updated with new information
- The probability table is small and efficient to handle
- The localisation is coarse (location oriented)

Pose estimation with Gaussian Model

- The pose is approximated by a single Gaussian function

$$p(s) = \frac{1}{\sqrt{2\pi\Sigma_s}} \exp\left(-\frac{1}{2}(s-\bar{s})\Sigma_s^{-1}(s-\bar{s})^T\right)$$

- s is here a continuous function and Σ_s is the associated uncertainty estimate
- Updating is normally performed using a Kalman filter model

Kalman filter – State space model

$$
\begin{aligned}
s_t &= F s_{t-1} + G u_t + w_t \\
z_t &= H s_t + v_t
\end{aligned}
$$

- where F is the system model, G is the deterministic input, H is a prediction of where features are in the world, w is the system noise, and v is the measurement noise

Monte-Carlo Based Methods

- Monte-Carlo based methods is using a sample model for approximation of the pose estimate
- Using a grid model as presented earlier
- Assume with have a number of particles in a collection

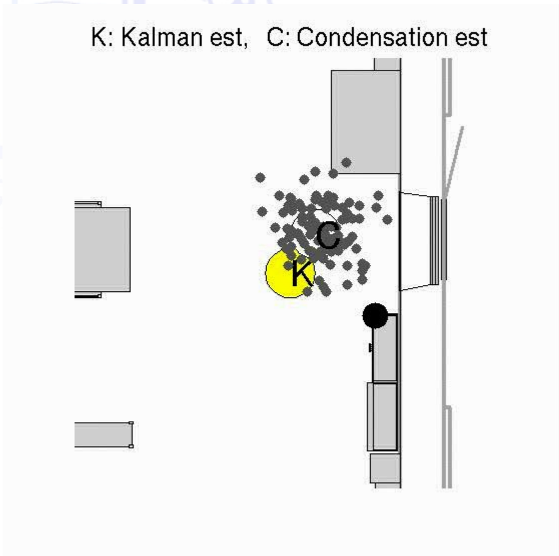
$$S_t = \left\{ (s_t^{(i)}, \pi_t^{(i)}) | i = 1..N \right\}$$

  each particle is a hypothesis for the position of the robot, and $\pi_t^{(i)}$ is an associated weight
- We can now approximate $p(s_t | z_0, z_1, ...z_t)$ for any distribution of the pose hypotheses

Monte-Carlo Strategy

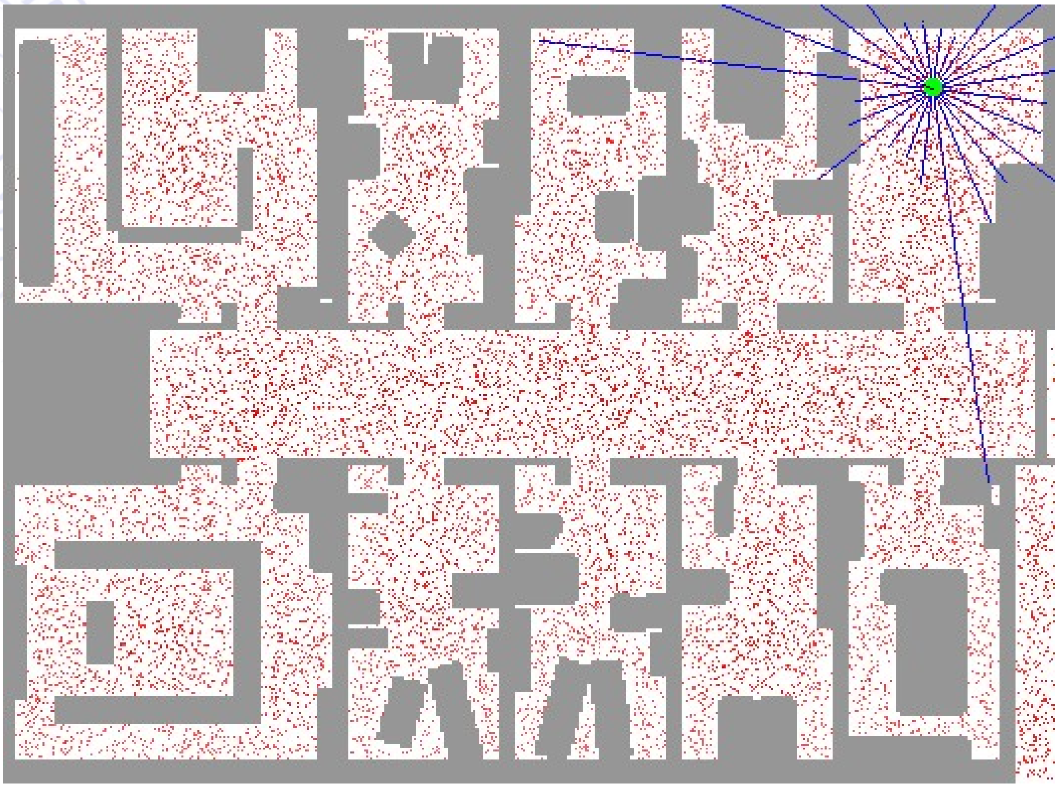
1. Draw N samples from an initial PDF. Typically a uniform distribution. Give each sample a weight of $\frac{1}{N}$
2. Propagate the motion information and draw a new sample from the distribution $p(s_{t+1}^{(i)}|s_t^{(i)}, o_t)$
3. Set the weight of the sample to $\pi_{t+1}^{(i)} = p(z_{t+1}|s_{t+1}^{(i)}) * \pi_t^{(i)}$ based on sensory input
4. Generate a new sample set by drawing samples from the current set and a basis distribution (typically uniform). Normalize the weights
5. Go back to step 2

Monte-Carlo Example



- Example of particle distribution about estimate of position
- Sonar readings for update of the position
- Video of system in operation

Monte-Carlo Example – Burgard, Fox & Thrun



Monte-Carlo Discussion

- Efficient to approximate any distribution of the pose
- The number of particles can be adopted to a particular platform
- Can be used both for simple and multi robot localisation

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Localisation / Mapping Example

- Now mapping and localisation is also integrated to allow for autonomous operation in general environments
- The mapping and localisation can be integrated to generate – Simultaneous Localisation and Mapping (SLAM)
- Indoor example VIDEO
- Outdoor example VIDEO

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Wrap-Up

- Localisation is a fundamental competence in mobile robotics
- Involves two major steps
  - Prediction of motion (kinematic modelling)
  - Updating of pose estimate(s)
- The method used depends upon the adopted model for handling of uncertainty and the associated world model
- Brief introduction to the main methods for estimation
- A few illustrative examples